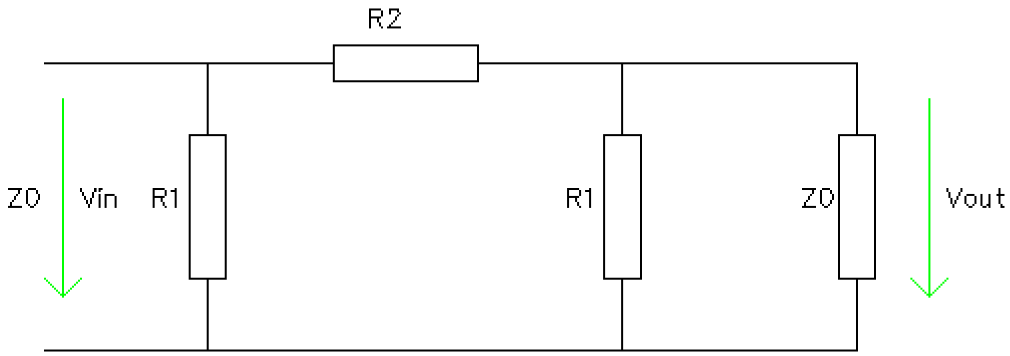
Deriving PI- attenuator formulae




$$V_{out} = \frac{R_1 \parallel Z_0}{R_1 \parallel Z_0 + R_2} V_{in} \tag{1}$$

$$A = \frac{V_{out}}{V_{in}} = \frac{R_1 \parallel Z_0}{R_1 \parallel Z_0 + R_2}$$

$$R_1 \parallel (R_1 \parallel Z_0 + R_2) = Z_0 \ \rightarrow R_1 \parallel \frac{R_1 \parallel Z_0}{A} = Z_0 \tag{2}$$

Solve $R_1$ from (2)

$$\frac{R_1\left(\frac{R_1 Z_0}{(R_1 + Z_0)A}\right)}{R_1 + \left(\frac{R_1 Z_0}{(R_1 + Z_0)A}\right)} = \frac{\frac{{R_1}^2 Z_0}{\cancel{(R_1 + Z_0)A}}}{\frac{R_1 A(R_1 + Z_0) + R_1 Z_0}{\cancel{(R_1 + Z_0)A}}} = \frac{{R_1}^{\cancel{2}} Z_0}{\cancel{R_1}(R_1 A + A Z_0 + Z_0)} = Z_0 \rightarrow$$

$$R_1 Z_0 = R_1 A Z_0 + A{Z_0}^2 + {Z_0}^2 \rightarrow R_1 \cancel{Z_0} - R_1 A \cancel{Z_0} = {Z_0}^{\cancel{2}}(A+1) \rightarrow R_1(1-A) = Z_0(A+1) \rightarrow$$

$$\boldsymbol{R_1 = Z_0 \frac{(1+A)}{(1-A)}} \tag{3}$$

Substitute $R_1$ in (2) with (3)

$$Z_0 \frac{(1+A)}{(1-A)} \parallel \left[ Z_0 \frac{(1+A)}{(1-A)} \parallel Z_0 + R_2 \right] = Z_0 \rightarrow$$

$$Z_0\frac{(1+A)}{(1-A)} \parallel \left[\frac{Z_0{}^{\cancel{2}}\frac{(1+A)}{(1-A)}}{\cancel{Z_0}\left(1+\frac{(1+A)}{(1-A)}\right)}+R_2\right] = Z_0 \rightarrow$$

$$Z_0\frac{(1+A)}{(1-A)} \parallel \left[\frac{\frac{Z_0(1+A)}{\cancel{(1-A)}}}{\frac{2}{\cancel{(1-A)}}}+R_2\right] = Z_0 \rightarrow$$

$$Z_0\frac{(1+A)}{(1-A)} \parallel \left[\frac{Z_0(1+A)+2R_2}{2}\right] = Z_0 \rightarrow$$

$$\frac{Z_0\frac{(1+A)}{(1-A)}\left(\frac{Z_0(1+A)+2R_2}{2}\right)}{Z_0\frac{(1+A)}{(1-A)}+\left(\frac{Z_0(1+A)+2R_2}{2}\right)} = Z_0 \rightarrow$$

$$\frac{\frac{Z_0(1+A)Z_0(1+A)+Z_0(1+A)2R_2}{\cancel{2(1-A)}}}{\frac{2Z_0(1+A)+Z_0(1+A)(1-A)+2R_2(1-A)}{\cancel{2(1-A)}}} = Z_0 \rightarrow$$

$$2R_2Z_0(1+A)-2R_2Z_0(1-A) = 2Z_0{}^2(1+A)+Z_0{}^2(1+A)(1-A)-Z_0{}^2(1+A)^2 \rightarrow$$

$$2R_2\cancel{Z_0}\left(1+A-(1-A)\right) = Z_0{}^{\cancel{2}}\left(2(1+A)+(1+A)(1-A)-(1+A)^2\right) \rightarrow$$

$$\cancel{2}R_2(2A) = \cancel{2}Z_0(1-A^2) \rightarrow$$

$$\boldsymbol{R_2} = \frac{\boldsymbol{Z_0\left(1-A^2\right)}}{\boldsymbol{2A}} \tag{4}$$